A3_7 Pandora's Floating Mountains: Lagrange Point

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November 27th, 2014

Abstract

This article investigates the physical possibility of the floating mountains depicted in the film "Avatar". It was theorised that should the L1 Lagrange point between the moon the movie was based on, and the gas giant it is shown to orbit lie on the surface of the moon, then any matter at that point should experience no net force and float. It was found that the moon would have to orbit very close to the gas giant, within 0.7151 planetary radii of the surface, and would therefore be unfeasible in the manner depicted in the movie, due to the natural disasters this would incur.

Introduction

The James Cameron movie "Avatar" features a mountain range dubbed the "Hallelujah Mountains", which are a set of very large boulders to mountains which apparently levitate or float completely unaided by any obvious means of propulsion or flight. A previous article has covered the possibility of this phenomenon occurring due to intense magnetism [1]. This article covers the extent to which the large gas giant planet, which the featured moon the film occurs upon orbits, could cause the phenomenon depicted in the movie. Between any two bodies in space, there exists a Lagrange point, where the gravities of the two bodies cancel out and there is no net force of gravity resulting from the two objects involved. If the gas giant is of sufficient size, and is close enough, the Lagrange point of the system could lie on the surface of the moon. This would cause any object at this point to experience a zero gravity field, allowing a mountain of any size to float.

Theory

The L1 Lagrange point is the one which falls between two bodies in space. Therefore, the conditions for its position are the result of the linear combination of the gravity potentials of the respective bodies:

$$\frac{GM_m m}{r_m^2} = \frac{GM_p m}{(r_s + r_p)^2}, (1)$$

$$\frac{M_m}{r_m^2} = \frac{M_p}{(r_s + r_p)^2}, (2)$$

$$r_s = r_m \sqrt{\frac{M_p}{M_m}} - r_p, (3)$$

where M_m denotes the mass of the moon, and r_m its radius; M_p is the mass of the planet it orbits, and r_s is the distance between the surfaces of the moon and planet, and r_p the radius of the planet.

For the moon to exhibit the Earth-like conditions depicted, it was assumed that it had a similar mass and density profile to that of the Earth [2], $r_m = 6371\text{km}$ and $M_m = 5.972 \times 10^{24}\text{kg}$. Likewise, the gas giant was assumed to have attributes akin to the gas giant Jupiter [3], $r_p = 69{,}910\text{km}$ and $M_p = 1.898 \times 10^{27}\text{kg}$.

Results

This gives $r_s = 4.367 \times 10^{7}\text{m} = 43{,}670\text{km}$, and an orbital radius (centre of planet to centre of moon, $r_o = r_m + r_s + r_p$) of $r_o = 119{,}900\text{km}$.

This puts the gas giant very close to the planet in question, less than 1 planetary radius of the gas giant ($0.7151r_p$), meaning that it would take up a significant portion of the sky. With the planet directly overhead, it would fill approximately

$$\frac{\theta}{2} = \tan^{-1}\left(\frac{r_p}{(r_s + r_p)}\right), (4)$$

$$\theta = 63.23°$$

of the sky. This is a significant coverage and, depending on its specific orbit, would cause the moon to be very often in the shadow of the planet, giving it a much more sporadic day/night cycle, and causing it to be in the dark far more often than if it were a separate planet, or in a much higher orbit.

Conclusion

While this radius of orbit is physically possible, the moon would suffer some major consequences from this orbit: the first being that the gas giant would leech the moon's atmosphere: as it extends to a higher altitude than the moon's surface and floating mountain ranges, the atmosphere would experience negative gravity in relation to the direction of the moon. Additionally, the moon would experience significant tidal forces, due to the gas giant's gravity. The moon Io [4], which orbits Jupiter, experiences tidal heating and regular massive volcanic eruptions due to the gravity effects from its host planet. The orbit of Io is just over 6 Jupiter radii in size, so this fictional moon should, supposedly suffer greater effects.

An additional problem is the moon's rotation. Either the moon must not rotate, so that the mountain range remains at the L1 Lagrange point; the moon rotates, and the mountains stay still, which would cause them to travel very quickly with respect to the reference frame of the moon's surface; or everything rotates, and every point along equator of the moon experiences a brief period of weightlessness every "day".

Therefore, while physically possible to float matter using the Lagrange points of orbits, the distances required would cause a reality far removed from the image depicted in the film "Avatar", with no atmosphere, huge tectonic instabilities, and sporadic weightlessness or very fast-moving mountains.

References

[1] *"Pandora's Floating mountains"; McQuade G, Walker M, Garland L, Bradley T; Journal of Physics Special Topics, Vol. 13 2014*

[2]http://en.wikipedia.org/wiki/Earth Last accessed 13/11/14

[3]http://en.wikipedia.org/wiki/Jupiter Last accessed 13/11/14

[4]http://en.wikipedia.org/wiki/Io_%28moon%29 Last accessed 27/11/2014